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Structural Change and Sustainable Development

by

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Abstract. In this paper, we show that the commonly observed decline in primary (natural resource using) sector output and employment shares can be explained as an endogenous response to the presence of nature's constraint. A decline in primary sector output and employment shares (often termed structural change) takes place even if consumer preferences are homothetic, and technological progress does not discriminate against the primary sector. Under certain conditions, structural change allows an open economy to grow with natural resource sustainability. Sustained and environmentally sustainable economic growth is possible even if the natural resource is exploited under open access. Well-defined property rights are neither necessary, nor sufficient for sustainable growth.

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I. Introduction

> "It has been said that the great question is now at issue, whether man shall henceforth start forwards with accelerated velocity towards illimitable, and hitherto unconceived improvement, or be condemned to a perpetual oscillation between happiness and misery, and after every effort remain still at an immeasurable distance from the wished-for goal." Thomas Malthus, *An Essay on the Principle of Population* (1798).

Are growth and environmental sustainability compatible? At least since Malthus, economists have pondered the relationship between growth and natural resources. Many remain concerned that continuous growth will eventually place unsustainable demands on our natural resources. Others are more optimistic. They believe that ever improving technology will allow society to produce greater levels of output without continuously damaging natural resources. According to them, technical change will make growth and environmental sustainability compatible.

In this paper we revisit the question of environmentally sustainable growth in the context of an economy that produces two final goods, one of which uses a renewable natural resource in its production. The economy possesses the standard tradeoff between growth and sustainability. Growth imposes increasing demands on the natural resource, which threatens the survival of the resource, and correspondingly the feasibility of economic growth. We ask three main questions. First, given a distortion-free economy that uses a natural resource as factor of production, what makes sustainable growth possible? Second, if the natural resource is characterized by ill-defined property rights, is sustainable growth still possible? Third, is the long-run rate of economic growth affected by suboptimal environmental policy associated with ill-defined property rights? [1]

[1] Our questions originate from the two main differences between man-made, and natural capital. Firstly, unlike man-made capital, natural capital involves a constraint outside human control. In the case of a non-renewable resource, this constraint is a finite stock. In the case of a renewable resource this constraint is a natural rate of growth and finite carrying capacity. Secondly, property rights on natural resources are often poorly defined or enforced (Brown (2000) provides an intuitive explanation of why property right issues pervade natural resources).

Through our analysis, we present a fairly intuitive but novel explanation for sustainable growth: structural change. [2] The economy we consider is small and open. It produces two final goods, a good that is resource based and another one that is produced without using the resource, and three assets. One asset is specific to the non-resource sector ("physical capital"), another is specific to the resource-based sector (natural capital) and the third asset (human or knowledge capital") enhances the productivity of labor in both sectors. We assume human or knowledge capital is embodied in raw labor, which is freely mobile across sectors. Accumulation of knowledge (or technical change) is endogenous and equally enhances the productivity of labor in the production of both final goods.

We propose that economic growth can be sustainable when accompanied by commonly observed changes in the sectoral composition of output. Along the economy's growth path, a relative contraction of the natural resource-using (or damaging) sector, and a relative shift in the economy's input composition towards the input specific to the non-resource sector, lowers the demand on the natural resource. This allows the economy to expand its output while still preserving its natural resource.

We derive conditions for sustainable development in two scenarios. The first scenario reflects conditions where property rights are well-defined on all assets, and markets are competitive. This scenario can be modeled through the "benevolent social planner" parable. The planner implements optimal natural resource policy by controlling resource extraction, by allocating part of the savings to the investment in the protection and renewal of the natural resource, and by mandating optimal investment in all other assets. The second scenario reflects conditions where the natural resource is subject to ill-defined property rights, or equivalently that the resource is harvested

[2] Structural change refers to a reduction of output and employment shares of the primary (mostly natural resource intensive production like mining, forestry etc.) sector during the growth process. It is widely accepted as a consistent stylized fact of the modern economic growth process. Chenery (1960) and Kuznets (1957) present structural change

under open access, and there is no investment in the natural resource. However, competitive markets and well-defined property rights still prevail in all other assets. In this scenario, competitive markets will still lead to (privately) optimal investment decisions in all assets except the natural resource.

In both scenarios the model predicts a decreasing share of the primary (resource-based) sector's output in GDP. It also predicts a decreasing share of the total labor force employed in the primary sector, and unbalanced growth in assets, favoring the asset that is specific to the non-resource sector. Along the growth path, physical, and human capital grow at non-constant rates different from each other. That is, the model predicts a broader version of structural change than the standard interpretation: Changes in the composition of final outputs away from primary or resource-based sectors and changes in the composition of inputs toward those most intensively used by the non-primary sectors. These predictions are consistent with well accepted stylized facts. The model also predicts that labor productivity grows in both the resource and non-resources sectors.[3]

We find that endogenous structural change is a key source of sustainability. Under fairly general conditions, structural change, along with positive economic growth is both necessary and sufficient for sustainable development. Importantly, we find that property rights on the natural resource are neither necessary nor sufficient for sustainable economic growth. Property rights, promote a higher level of the natural resource, and allow the economy to achieve higher welfare in equilibrium. However, the absence of property rights (or equivalently of environmental policy) does not threaten the feasibility of economic growth. As long as other assets have well-defined property

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(both across countries and over time) as a stylized fact of the modern growth process. Kongsamut et. al. (2001) also provides evidence of structural change.

[3] Productivity in the primary or resource sectors has experienced significant growth over the long-run (Parry, 1999). New technologies are rapidly incorporated into the resource sectors. In fisheries for example, the use of radar, sonar, track plotters and electronic navigation aids have significantly increased the productivity of fishing effort. In agriculture, new breeds and genetic improvement, together with improving irrigation techniques and better machinery constantly raise productivity in the sector. In forestry, the use of information technology and improved capital equipment has also substantially raised productivity in the forestry sector.

rights and all markets are perfectly competitive, sustained and environmentally sustainable economic growth remains feasible.

The intuition behind our results is fairly simple. Structural change in this model is an endogenous response to the constraint imposed by nature. Unlike man-made capital, the natural resource is limited to its carrying capacity. This implies that even if technical change has identical impacts across sectors, the growth in labor productivity in the natural resource sector cannot match the growth in productivity in the non-resource sector where all assets are man-made and expanding over time. For this reason, investment in all forms of capital adjusts to maximize the growth in output in society. In other words, the technical change observed in our model endogenously responds to the scarcity of natural capital.[4]

Somewhat surprisingly, even if there are no property rights on the natural resource, the endogenous adjustment in the structure of output persists. Under open access, there is no investment in the natural asset, and the natural resource is over-harvested. However, despite these problems, labor market interactions preserve structural change, and sustainability. Given well functioning markets elsewhere, increases in man-made capital still imply a divergence of productivity across the two sectors. To maintain equilibrium in the labor market, labor is drawn towards the non-resource sector. Open access does not threaten the means to sustainability; it only raises the amount of labor employed in the resource sector at any given time. This increase implies that the equilibrium stock of natural resource is lower. Open access also implies that the equilibrium welfare is lower as rents from harvesting are completely dissipated.

[4] In the absence of structural change, output would expand equally in both sectors. This would require that the extraction of the natural resource also expands at the rate of growth in man-made assets. Given a natural limit, this would draw down the natural asset, and makes it relatively scarce in comparison to man-made assets. A divergence in scarcity would imply a corresponding divergence in asset values. Clearly, this cannot be an equilibrium situation. The only way to maintain equilibrium in the presence of a natural resource is for the natural resource using (or damaging) sector to contract in response to the limit on the natural asset. Note that the equilibrium proposed is a straightforward application of Hotelling's rule (Hotelling (1931)). According to Hotelling, in equilibrium the natural asset is equally valuable as all other assets in society.

The result that optimal environmental policy is not necessary for sustainable growth is in sharp contrast with the findings by the literature focusing on pollution rather than natural resources, in which optimal environmental policy is a necessary condition for sustainable growth (Stokey, 1998; Aghion and Howitt, 1998). The reason is that this literature assumes that pollution only affects the utility function but it has no feed-back (negative) effects on the productivity of the production sector. Thus, in the absence of environmental policy that imposes a cost to pollution emissions, firms have no incentives to restrain emissions and, therefore, sustainable growth is infeasible. By contrast, when the environment is also a factor of production as in our case, as the natural resource becomes scarcer, firms have incentives to substitute and to shift toward sectors that do not use the natural resource.

Our paper is related to two distinct branches of the literature. First is the literature on sustainable growth in the presence of natural resources or pollution.[5] The literature on sustainable growth is, with a few exceptions, almost entirely based on models with a single final good. In a model with a single final good, the growth of income and environmental damage derives from the same sector. More often than not, this implies that sustained and environmentally sustainable growth is possible only after making special assumptions. Smulders and Gradus (1996) provide restrictions on production and pollution abatement technologies that make growth environmentally sustainable. Their restrictions are, a unitary elasticity of substitution between pollution and physical capital in production, which as we see later is also shared by our model, and a greater than unitary elasticity of pollution reduction from abatement expenditures, an assumption that we do not need.

[5] This literature consists largely of recent economic exchanges grounded in neoclassical growth theory. It was in part triggered by the publication of Donella H. Meadows, Dennis L. Meadows, Jorgen Randers & William W. Behrens III (1972) *Limits to Growth*, Potomac Associates, New York. See Brock and Taylor (2004), and Smulders (1999) for excellent surveys.

In other words, abatement expenditures are relatively more effective in reducing pollution than increases in pollution from growth.[6] [7]

Stokey (1998) proposes exogenous technical change as the reason for sustained and environmentally sustainable growth. Aghion and Howitt (1998) extend the Stokey (1998) model demonstrating that sustainable growth is possible without exogenous technical change, or exogenously imposed externalities. However, they remain concerned with the assumption that the elasticity of marginal utility of consumption has to be greater than one (or that utility is highly concave), which is necessary in both Stokey's and Aghion and Howitt's models to attain sustainable growth.[8] In a model with homothetic preferences, it ensures that individuals are willing to make the required sacrifices in consumption needed for sustainability.

As indicated earlier, the fact that Aghion and Howitt (1998) and Stokey (1998) focus on pollution affecting consumer utility, while we focus on natural resources as a factor of production has drastic implications for the likelihood of sustainable development. Unlike them, we neither need optimal environmental policy nor to impose the assumption of a greater than unitary elasticity of the marginal utility as necessary conditions for sustainable development.[9]

Perhaps the work most closely related to ours includes recent papers by Eliasson and Turnovsky (2004) and Carol McAusland (2003). These papers are also concerned with sustainable and endogenous growth in a two-sector small open economy with the renewable natural resources as a factor of production. Our paper differs from Eliasson and Turnovsky in several respects.

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[6] In contrast, our model does not impose any asymmetry in the effect of harvesting (growth) and man-made investment in the natural asset (abatement).

[7] Another popular special assumption is externalities. Huang and Cai (1994) posit government spending externalities on pollution control as a reason for sustainable growth. Schou (2000), meanwhile, posits human capital externalities as the source of sustainable growth.

[8] Aghion and Howitt (1998) summarize their model of renewable natural resources in the following words, "it appears that unlimited growth can indeed be sustained, but it is not guaranteed by the usual sorts of assumptions that are made in endogenous growth theory" (p. 162).

[9] We believe that conditions sufficient for sustainable growth in the absence of utility benefits, are also likely to be sufficient for sustainable growth when the resource produces utility in addition to output. This is because the important

Firstly, we do not rely on capital accumulation externalities as they do to induce productivity growth. Productivity in our model grows due to endogenous increases in human or knowledge capital. Secondly, while there is structural change in the model by Eliasson and Turnovsky, their structural change is limited to changes in the composition of the value of output. There is no structural change in the composition of inputs employed. Further, structural change in their model is the result of the assumption that productivity grows only in the non-resource industry (due to the presence of an externality) while any productivity growth in the resource sector is *a priori* ruled out. In contrast, in our model structural change is an endogenous response brought about due to an unbalanced growth in assets and labor market interactions despite that labor-augmenting technical change takes place at the same rate in both the resource and non-resource sectors.

McAusland (2003), in a two sector model, but with only two inputs, relies on learning-by-doing externalities to generate economic growth. Further, she focuses exclusively on the case of open access, and then studies the possibility of sustainable growth in both a closed and open economy. In contrast, we consider the open economy case, but study sustainable growth under well-defined property rights, and under ill-defined property rights or open access.

Our paper is also related to the literature on multi-sector growth models explaining structural change. Previous analyses have explained structural change largely as the result non-homothetic preferences. As consumers place a higher value on certain types of goods on becoming wealthier, the response of a freely functioning economy involves a change in its structure to reflect the changes in demand (Echevarria (1998), Laitner (2000) and Kongsamut et al. (2001)). In addition to preferences, Baumol (1967) and Baumol et al. (1985) also highlight the importance of imbalanced technical change. By contrast, our paper shows that structural change is likely to take place even if preferences are homothetic and technical change is balanced. We argue that given a constraint on

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thing is to be able to maintain a stable stock of natural resources and prevent extinction. If this is possible with only benefits in output, it is also likely to be possible when consumers value natural resources.

the size of the natural asset, a relatively well functioning economy experiences an adjustment in assets, and subsequently an adjustment in outputs, which biases growth towards its most productive sectors (which involves a decline in the relative share of the primary sector).

The remainder of the paper is structured as follows. In Section II we present the model of a small open economy used for our analysis. In Section III we define long-run equilibrium for our model, and present an analysis of the properties of this long run equilibrium. In Section IV we examine the consequences of the absence of an optimal environmental policy. In section V we discuss the transitional dynamics of the model, and conclude in Section VI.

II. The Model

Consider a small open economy with a representative agent consuming two final goods (one 'clean' and one 'dirty'). The production of the *clean* good has no impact on the environment, while production of the *dirty* good harms the environment. The dirty good represents all natural resource-intensive (harvesting or extractive) sectors of the economy. There are three assets in which the economy can invest: human capital, natural capital, and physical capital.

Consumption

The economy is assumed to be small and open. This implies that prices are exogenously given. Let x_c denote consumption of the clean good and x_d denote consumption of the dirty good. Preferences of the representative consumer are given by,

$$U(x_c, x_d) = \frac{\varepsilon}{\varepsilon - 1}[v(x_c, x_d)]^{\frac{\varepsilon-1}{\varepsilon}}, \qquad (1)$$

where, $\varepsilon > 0$ is a fixed parameter representing the elasticity of inter-temporal substitution or the reciprocal of the elasticity of marginal utility, and $v(x_c, x_d) = x_c^{\xi} x_d^{\psi}$ is an index of aggregate consumption, with parameters $\xi > 0, \psi > 0$. It is natural to assume that the consumption aggregator, v, is homogenous of degree one, which means $\xi + \psi = 1$.

Production and Labor Market Clearing

Inputs used in the production of the clean sector are raw labor (l_c), labor augmenting knowledge or human capital ($h \geq 1$), and sector-specific non-human or "physical" capital (k). The production of the clean good is given by

$$y_c = F\left(k, hl_c\right), \tag{2}$$

where $F(\cdot)$ is concave, increasing and homogenous of degree one in its inputs. We also assume that $F(\cdot)$ satisfies the *Inada conditions*, in other words, as the quantity of an input approaches zero its marginal product tends to infinity. To illustrate our model, we sometimes use a standard Cobb-Douglas form for our production function

$$F\left(k, hl_c\right) = Ak^{\alpha}\left(hl_c\right)^{1-\alpha}, \tag{3}$$

where A is a positive constant, and $0 < \alpha < 1$ is the constant share of capital in total production.

The dirty sector includes production that is directly dependent on natural resources. Examples are: primary industries such as mining, fishing, forestry, and agriculture. Production of the dirty good uses raw labor (l_d) and the stock of natural capital (n) as inputs. The dirty or resource sector also benefits of labor-augmenting technical change, h.

The production function for the dirty good is

$$y_d = nhl_d. \tag{4}$$

This function corresponds to the standard specification for production based on natural resources. It was introduced to the literature in a more general form by Gordon (1954) who argued that the law of diminishing returns was not appropriate for modeling fisheries. The specific form in (4) above was proposed later by Schaefer (1957). Since then this specification has been widely used in modeling natural resources (see Brander and Taylor (1998 and 1998a), Conrad (1995), and Munro and Scott (1993) for examples). Every unit of dirty good output reduces current stock of natural capital by ϕ. If the dirty good is assumed to be a primary or extractive commodity, output (from equation(4)) can be written as $y_d = [\phi h l_d + (1\text{-}\phi) h l_d] n$, where $\phi n h l_d$ is the harvest of the resource, and $(1\text{-}\phi) n h l_d$ is processing. It is natural to assume that $0 < \phi < 1$, in other words producing one unit of the dirty good reduces the natural capital by less than one unit.

The labor market is perfectly competitive. Aggregate labor supply in the economy is fixed at $\bar{L}$. Labor market clearing requires the sum of labor employed in the dirty and clean sectors to equal the total labor supply, $l_c + l_d = \bar{L}$.

Asset Accumulation

Let I_j denote investment in each of the assets $j \in \{h, n, k\}$. Growth in labor augmenting human (knowledge) capital is given by

$$\dot{h} = I_h - \delta_h h, \qquad (5)$$

where δ_h is the rate of depreciation of h and $I_h \geq 0$. The rate of depreciation of knowledge can be interpreted as the proportion of annual retirements from the workforce. Any growth in h augments the effective labor input in both sectors (see equations (3) and(4)). In other words, growth in knowledge equals endogenous labor augmenting technical change. The non-negativity constraint for I_h is quite natural, indicating that labor-augmenting technology can be expanded but not reduced except through natural depreciation over time.

Let $g(n)$ be the intrinsic growth function of the renewable natural resource. The function has an inverted U shape with $g(0) = g(\bar{n}) = 0$, where $\bar{n}$ is the 'carrying capacity' of the natural resource. The *carrying capacity* of a natural resource is the maximum stock that can be sustained in its natural surroundings.[10] The function is assumed to be concave. To illustrate we will occasionally use the logistic function, the most commonly applied to natural resources:

$$g(n) = \gamma n(1 - n/\bar{n}), \qquad (6)$$

where γ is the maximum or *intrinsic* growth rate of the stock. Let I_n denote human investment in natural capital, such as: tree planting, national park protection, fish replenishment including aquaculture investments, protection of marine ecosystems, soil protection including terracing, drainage, agricultural fallowing as well as the cleaning-up of ecosystems. Evolution over time of the natural resource stock is

$$\dot{n} = \begin{Bmatrix} g(n) + I_n - \phi nhl_d, \; if\ \bar{n} \geq n \geq 0 \\ -\phi nhl_d, \qquad if\ \ n > \bar{n} \end{Bmatrix}. \qquad (7)$$

Growth of the natural resource comprises its natural capacity to regenerate ($g(n)$), investment in expanding natural capital (I_n), and the reduction of natural capital from production of the dirty good (ϕnhl_d). The non-negativity constraint also applies to investment in natural resource, $I_n \geq 0$ (efforts will not be spent in reducing the stock of the natural resource except through its more intense extraction). Also, note that investment cannot maintain the natural resource beyond its carrying capacity. Investment can substitute for natural regeneration only if the natural resource is within its natural bounds. This reflects the fact that a natural resource involves constraints outside human control. If investment in the natural resource could maintain the stock beyond its natural carrying capacity, a natural resource would be no different from other forms of man-made capital.

The stock of physical capital k grows according to the following equation.

$$\dot{k} = I_k - \delta_k k, \qquad (8)$$

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[10] If there is no extraction, the stock of natural resource stabilizes at its carrying capacity (see Conrad (1995) for other examples of such a growth function).

where δ_k is the rate of depreciation of k and $I_k \geq 0$ is the investment in physical capital also subject to a non-negativity constraint.

The Social Planner's Problem

The social planner maximizes the present value of utility for the representative consumer by optimally investing in human, natural and physical capital. The social planner can be interpreted as a normative benign dictator who sets all variables to optimal levels conditional on exogenous variables. Alternatively, the social planner's solution can be considered as the positive outcome of an economy where all markets (including resource extraction) function competitively with perfectly defined property rights[11]. Through the course of this paper we often use the positive interpretation for the social planner's solution.

Formally, let ρ denote the social discount rate. The social planner's maximization problem is

$$V \equiv \max_{x_c, x_d, l_c, I_h, I_n, I_k} \int_0^\infty U\left(x_c(t), x_d(t)\right) \exp^{-\rho t} dt \qquad (9)$$

where t denotes time. Maximization of utility is subject to the following set of constraints: all capital growth equations ((5)-(8)), initial conditions for capital stocks, (for human capital $h(0) = h_{00}$ where $h(t)$ is the stock of human capital at time t, for natural capital $n(0) = n_{00}$, and for physical capital $k(0) = k_{00}$), non negativity constraints $x_c \geq 0$, $x_d \geq 0$, $l_c(t) \geq 0$, $I_h(t) \geq 0$, $I_n(t) \geq 0$, and $I_k(t) \geq 0$, and the following budget constraint or current account equilibrium,

$$x_c + px_d + I_h + I_n + I_k \leq pnh\left[\bar{L} - l_c\right] + F\left(k, hl_c\right). \qquad (10)$$

The budget constraint requires that total consumption and total investment expenditures should be no greater than society's total income.

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[11] This equivalence has its roots in the first and second fundamental theorems of welfare economics (see Arrow and Debreu (1954) and the references included therein). Since our key concern is the resource sector, we later remove the assumption of perfect property rights upon the natural resource while retaining the assumption of well-defined property rights for all other factors and that markets operate under perfect competition

It is important to note that given a small open economy this budget constraint also requires that import expenditures cannot exceed the value of exports. If the budget constraint holds with equality (which occurs given our maximization assumption), it necessarily implies balanced trade and vice-versa (Dixit and Norman (1980))[12].

We assume that the clean good is used for investment (this is reflected in equation(10), where investments in assets are priced at unity). Note, however, that this does not imply that all revenues from the resource good are consumed. Such revenues can be used to import investment goods. In fact, specifying the current account in (10) is quite flexible in several respects. Firstly, it allows either production at home or the import of any consumption or investment goods. Secondly, it allows the outputs from each sector to either be exported or imported. Thirdly, it allows revenues from each sector to be used either for financing domestic or imported goods for consumption or investment.

Let λ be the Lagrangian multiplier associated with the budget constraint, and μ, η and Ω be the co-state variables associated with human, natural, and physical capital, respectively.[13] The solution to the problem in equation (9) is found by maximizing a continuous time current value Hamiltonian,

$$\begin{aligned} \mathrm{H} = {} & U\left(x_c, x_d\right) + \lambda\left[F\left(k, hl_c\right) + pnh\left(\overline{L} - l_c\right) - x_c - px_d - I_h - I_k - I_n\right] \\ & + \mu\left[I_h - \delta_h h\right] + \Omega\left[I_k - \delta_k k\right] + \eta\left[I_n + g\left(n\right) - \phi nh\left(\overline{L} - l_c\right)\right] \end{aligned}, \qquad (11)$$

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[12]From the optimization problem it is clear that the budget will be exhausted and therefore (9) holds as equality. In this case we can rewrite (9) as $p\left(nh(\overline{L} - l_c) - x_d\right) = x_c + I_h + I_n + I_k - F(k, hl_c)$. That is, any excess supply (net exports) of one of the goods is exactly equal to the excess demand (net imports) of the other valuing the goods at the world prices. This of course implies that the trade account is balanced. An advantage of this general formulation of the trade balance over other more specific formulations of the balanced trade condition is that we do not *a priori* define which good is exported and which one is imported. *This is an endogenous outcome of the model solution.*

[13] The fact that the shadow price of natural resources, η is at the optimal level implies that the there are perfect property rights on the natural resource. Equivalently, it implies that the planner is fully internalizing the environmental externalities. Later we shall consider the case where η does not play a role in the solution. In other words, we shall relax the assumption of perfect property rights on the natural resource.

where H is defined under the assumption that the natural resource is within its natural bounds $n \in (0, \bar{n})$.

The first order conditions for the Hamiltonian are given below (subscripts on functions denote partial derivatives). The first order conditions with respect to the two consumption goods are

$$\xi (x_c^{\xi} x_d^{\psi})^{\frac{\varepsilon-1}{\varepsilon}-1} (x_c^{\xi-1} x_d^{\psi}) = \lambda \tag{12}$$

$$\psi (x_c^{\xi} x_d^{\psi})^{\frac{\varepsilon-1}{\varepsilon}-1} (x_c^{\xi} x_d^{\psi-1}) = \lambda p \tag{13}$$

Using (12) and (13) it immediately follows that at the optimum x_c and x_d are consumed in fixed proportion for a given level of *p*,

$$x_d = \frac{\psi}{\xi p} x_c . \tag{14}$$

Using (14) in (12) we can represent the consumption optimality condition in terms of the marginal utility of x_c only:

$$\xi \left(\frac{\psi}{\xi p} \right)^{\frac{\psi(\varepsilon-1)}{\varepsilon}} x_c^{-\frac{1}{\varepsilon}} = \lambda \tag{15}$$

Since x_c and x_d are proportional for given *p*, we have that the rate of growth of U is entirely determined by the rate of growth of x_c.

The Kuhn-Tucker first order conditions with respect to investment in human, physical and natural capital are:

$$-\lambda + \mu \le 0,\ I_h [-\lambda + \mu] = 0,\ I_h \ge 0 , \tag{16}$$

$$-\lambda + \Omega \le 0,\ I_k [-\lambda + \Omega] = 0,\ I_k \ge 0 \text{, and} \tag{17}$$

$$-\lambda + \eta \le 0,\ I_n [-\lambda + \eta] = 0,\ I_n \ge 0 . \tag{18}$$

The condition for the optimal allocation of labor between the two sectors is,

$$\lambda\left[F_2(\cdot)h - pnh\right] + \eta\phi nh = 0. \tag{19}$$

Conditions (12) and (13) imply that the marginal values of consumption should equal the shadow value of income, λ (which is positive given positive marginal utilities). Conditions (16) to (18) are Kuhn-Tucker conditions indicating that positive investment in any of the assets must imply that the shadow value of such asset is equal to the shadow value of income or consumption. If an asset's shadow value is below the shadow value of income there will be zero investment in such an asset. These conditions also imply that the assets' shadow value can be no greater than the marginal value of income, λ.

Condition (19) reflects efficiency in the labor market. At each point in time, the value of the marginal product of labor in the clean sector, $\lambda F_2(\cdot)h$, should equal the value of the net marginal product of labor in the dirty sector, $(\lambda p - \eta\phi)nh$. The value of the net marginal product of labor is, in turn, equal to the marginal value product of labor in producing the dirty output (λpnh) minus the value of the resource degradation caused by such marginal output ($\eta\phi nh$). Importantly, since the labor market must clear at all points in time, this condition must be satisfied at all points of time as well.

Now consider the co-state variable dynamics for each of the three assets:

$$\dot{\mu} = (\rho + \delta_h)\mu - \lambda\left(F_2(\cdot)l_c + pn(\bar{L} - l_c)\right) + \eta\phi n(\bar{L} - l_c) \tag{20}$$

$$\dot{\Omega} = \left(\rho + \delta_k\right)\Omega - \lambda F_1(\cdot) \tag{21}$$

$$\dot{\eta} = \left(\rho - g_n\right)\eta - h\left[\bar{L} - l_c\right]\left(\lambda p - \eta\phi\right). \tag{22}$$

Finally the transversality conditions for this model are,

$$\lim_{t\to\infty} e^{-\rho t}\mu h = 0,$$
$$\lim_{t\to\infty} e^{-\rho t}\Omega k = 0, \tag{23}$$
$$\lim_{t\to\infty} e^{-\rho t}\eta n = 0.$$

III.- Long Run Growth

Differentiating (12') with respect to time we obtain the usual result that the rate of growth of x_c must be directly related to the negative of the rate of growth of λ (the marginal utility of consumption)

$$\hat{x}_c = -\varepsilon\hat{\lambda}\,, \qquad (24),$$

where we use a 'hat' to denote rate of change, i.e. $\hat{x}_c \equiv \dot{x}_c / x_c$ (note from (14) that $\hat{x}_c = \hat{x}_d$). Given that $\hat{\lambda} < 0$ (from declining marginal utility of consumption) any $\varepsilon > 0$ is consistent with growing consumption. That is, growth does not require any restriction on the inter-temporal elasticity of substitution (and, consequently, on its reciprocal, the elasticity of the marginal utility) other than being positive.

Before we continue any further, it is useful to define the sustainable growth equilibrium.

Definition. We say that the planner's problem achieves *long-run sustainable growth equilibrium* (LSGE) when the rate of growth of consumption is positive and constant over time and the level of the resource stock is constant, that is $\dot{n} = 0$.

This definition of LSGE is fairly general. Note that given a positive rate of growth of consumption, the rate of growth of welfare is also constant and positive in LSGE. Thus, a positive growth in welfare combined with a constant stock of natural capital constitutes sustainable growth. We shall explore the implications of LSGE in Propositions 1 and 2 below. Then in Proposition 3 we shall present conditions under which the LSGE is feasible.

Proposition 1. *In a LSGE: (a)* $\hat{\lambda} = \hat{\mu} = \hat{\Omega} > 0$*; and (b)* λ / η *is constant. Consequently,* $\hat{\lambda} = \hat{\mu} = \hat{\Omega} = \hat{\eta}$.

Proof: See Appendix A.

Proposition 1 demonstrates that in a LSGE all shadow values should decrease at the same rate and that the shadow values of income, human capital and physical capital should be identical. These results on the shadow values help us further characterize the solution for LSGE. We can rewrite the labor market equilibrium condition (equation (19)) to get,

$$F_2\left(k / hl_c, 1\right) = (p - r\phi)n \tag{25}$$

where $r \equiv \eta / \lambda$ is a constant. Further, by setting $\hat{\Omega}$ and $\hat{\eta}$ equal to each other (using equations (20) and (21)) we obtain,

$$F_1(k / hl_c, 1) - \delta_k = g_n(n) + \left(p / r - \phi\right) h(\bar{L} - l_c). \tag{26}$$

In other words, the net marginal product of physical and natural capital is equal in LSGE. Next if we equalize $\hat{\mu}$ and $\hat{\Omega}$ (using (20), (21) and subsequently (25)) we obtain,

$$F_2(k / hl_c, 1)\bar{L} - \delta_h = F_1(k / hl_c, 1) - \delta_k. \tag{27}$$

The above equation implies that in the LSGE, net marginal products of k and hl_c in the clean sector equal each other. Note given our production function, there is a unique (k / hl_c) level at which these net marginal products are equalized. In other words, this expression implies that the (k / hl_c) ratio remains constant during the LSGE.

Given a constant (k / hl_c), equation (25) implies that there is a unique and constant stock of natural capital, n^*, in LSGE. Together a constant stock of natural capital, and a constant (k / hl_c) imply that $h(\bar{L} - l_c)$ also remains constant in the equilibrium (see Equation (26)).

We can summarize the relationships discussed in the above equations in Figure 1. The horizontal line represents equation (27) while the solid upward-sloping line represents equation (25), which is the equality among returns to labor between the two competing sectors. Since returns to labor in the clean sector are directly related to the level of natural resources, this latter line is positively sloped. The figure shows that if the level of natural resources n was higher (lower) than n^*, the returns to human capital (in both sectors) would be higher (lower) than the returns to physical capital which would be unsustainable in LSGE.

Note that given a constant stock of natural resources, the equation of motion of the resource stock (equation (7)) implies

$$g(n)/n + I_n/n - \phi h(\bar{L} - l_c) = 0. \qquad (28)$$

The system of equations (25) to (28) solves for the four endogenous variables: the effective capital to labor stock in the clean industry $(k/hl_c)^*$, the optimal stock of natural capital n^*, the effective labor employed in the dirty industry $[h(\bar{L} - l_c)]^*$, and the investment in natural capital $I_n^* \geq 0$ or $r^* \leq 1$.

Through the next proposition we investigate the implications of LSGE on investment in natural capital. In principle there are two possible solutions; the first is where $I_n^* > 0$ and therefore $r^* = 1$. Alternatively we can have $I_n^* = 0$, and from the Kuhn-Tucker conditions (18) $0 < r^* < 1$. In Proposition 2 we demonstrate that there is positive investment in natural capital while at LSGE.

Proposition 2*.* *In LSGE* $r^* = \dfrac{\eta}{\lambda} = 1$ *and, therefore,* $I_n^* > 0$.

Proof. See Appendix A.

Proposition 2 demonstrates that investment in natural capital is positive along the LSGE. This result also implies that the shadow value of natural capital equals marginal utility of consumption ($\eta = \lambda$), and $r^* = 1$. Using this result, the rate of growth in consumption (from equation (23')) along the LSGE can now be expressed as

$$\hat{x}_c = \varepsilon[F_2((k/hl_c)^*,1) - \rho - \delta_h] \tag{29}$$

Recall that since along the LSGE $h(\bar{L} - l_c)$ is constant, the primary sector remains stagnant over the long run. In other words, this growth in consumption derives purely from an expansion of the clean sector.

Through the next proposition we provide the feasibility conditions necessary and sufficient for LSGE.

Proposition 3. *An economy will achieve LSGE if and only if: (a)* $\rho < F_2((k/hl_c)^*,1) - \delta$ *; (b)* $\rho > [(\varepsilon - 1)/\varepsilon][F_2((k/hl_c)^*,1) - \delta]$*; (c)* $F_2((k/hl_c)^*,1)\bar{L} - \delta_h \geq g_n(n^*) + [(p-\phi)/\phi]g(n^*)/n^*$. *In addition, LSGE is* ***diversified*** *if (d)* $p > \phi$ *; and (e)* $F_2((k/hl_c)^*,1) < (p-\phi)\bar{n}$.

Proof. Please see discussion below and Appendix C.[14]

Conditions *(a)* and *(b)* are standard for most endogenous growth models. Condition *(a)* ensures that the economy is productive enough for growth to be possible (see equation (29)). It requires that the marginal productivity of the assets (human capital in this case) is greater than the discount rate (mathematically this condition also implies that $\hat{\lambda} = \hat{\mu} = \hat{\Omega} = \hat{\eta} < 0$). In the case where assets are not productive enough, the optimal path requires an unsustainable consumption path that leads to the implosion of the economy. Condition *(b)* results from the transversality conditions (please see Appendix C for a complete proof). This condition requires the discount rate to be larger than the rate of growth of welfare. If this condition does not hold the objective function is convex and welfare cannot be maximized.

[14] Appendix B provides a complete solution of LSGE with the specific functional forms proposed earlier.

Condition *(c)* is needed for Proposition 2 to hold, it provides the technological and natural conditions for $g(n^*)/n^* - \phi h(\overline{L} - l_c)^* \leq 0$, which implies that in LSGE $I_n^* \geq 0$ and $\eta = \lambda$, both of which are requires for a diversified equilibrium. In addition, this condition assures that the net marginal product of man-made assets in the clean sector is at least as large as the net marginal product of nature in the primary sector (that is, that equation (26) holds). Thus, condition *(c)* ensures that the clean sector can compete with the primary sector. Because there is a unique equilibrium ratio k/hl_c, the marginal products of the assets used by the clean sector are in fact constant. As a consequence of this, the Inada conditions are not sufficient to assure the survival of the clean sector. Note that since n^* is decreasing in p and since g_n and $g(n)/n$ are both decreasing in n, the right-hand-side of the inequality in *(c)* is increasing in p. This means that there is a critical level of the price of the dirty good above which the condition is not met. That is, if the productivity of the clean sector is too low or if the price of the dirty good is too high LSGE is not feasible. [15]

If condition *(c)* is not satisfied the economy specializes in the dirty sector. Specialization in the dirty sector, however, precludes LSGE. The reason is that natural capital cannot expand beyond the maximum carrying capacity, $\overline{n}$. That is, at some point economic growth must come to a halt. This occurs because, in contrast with the case of a diversified equilibrium, the expansion of h along the growth process can no longer be compensated by a reduction of l_d. As h increases natural resource extraction continuously expands, which requires a continuous increase in investment in the natural asset. However, given a limit of carrying capacity, this is not possible.

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[15] To illustrate condition *(c)* consider the specific functional forms from equations (3) and (6). Condition (c) under theses functional forms reduces to $p \leq \frac{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\overline{n}}\left[\frac{\phi}{\gamma}\overline{L}^{(1-\alpha)} + (2\phi+1)\overline{L}^{(1-\alpha)}\right] - \frac{\phi}{\gamma}\delta_h$. It is more likely that condition *(c)* holds if A, $\overline{L}$ are high, and it is less likely that condition *(c)* holds if p is high.

Conditions *(d)* and *(e)* are necessary to ensure the survival of the primary sector but are not necessary for LSGE. If $p < \phi$, the revenues from producing one unit of the dirty good are less than the cost in environmental damage from production. In other words the sector is not competitive. Given well-defined property rights, when $p < \phi$, the social and private returns to labor in the dirty sector, $(p-\phi)nh$ are negative. This implies that the sector cannot compete with the positive returns in the clean sector, and labor migrates completely to the clean sector.

However, despite the price of the dirty good being higher than environmental damage it is possible that the resource extractive sector is just not productive enough to be competitive (alternatively, the clean sector is comparatively too productive). Through condition *(e)* we rule out this possibility. If this condition is not met, the highest returns to human capital in the dirty sector, $(p-\phi)\overline{n}$, are not enough to compete with the clean sector. In such a case, the dirty sector must close.

If, either condition *(d)* or *(e)* is not satisfied then the system specializes in the clean sector. In this case the model yields the well-known two-asset balanced growth model of Chapter 5 in Barro and Sala-I-Martin (1995) as a special case. That is, the standard two-asset growth model with constant returns to scale can be regarded as a characterization of an economy where the non-primary sector is so productive that the primary sector is unable to compete. In this case LSGE is of course attainable and the stationary level of the resource is the maximum carrying capacity, $\overline{n}$.

Some Further Implications of the LSGE

From the equilibrium level of k/hl_c derived from (27) we have that $l_c = (k/h)\cdot c_1$, where c_1 is a positive constant that depends on the depreciation rates and parameters associated with the production function of the clean output. Equalizing the expressions for l_c arising from (27) and (26) and using $r = 1$, we obtain the following equilibrium relationship between *k* and *h*:

$$k = \frac{h\bar{L}}{c_1} - \frac{\chi(n^*)}{(p-\phi)c_1}, \tag{30}$$

where we use the positive constant $\chi(n^*) \equiv (p-\phi)n^*\bar{L} - \delta_h - g_n(n^*)$ to reduce notation clutter. Using equations (25) to (27) it immediately follows that $\chi = (p-\phi)h(\bar{L} - l_c) \geq 0$. The long-run relationship between *h* and *k* expressed in equation (30) is captured in Figure 2. Points above the positively sloped equilibrium line, represent combinations of (*k*,*h*) where physical capital has a higher marginal product than the natural capital. Since both $\chi(n^*)$ and c_1 are positive, the equilibrium line crosses the vertical axis at negative levels of *k*. This means that as *k* and *h* grow in equilibrium $\hat{k}$ must be larger than $\hat{h}$. In other words, we have asset-unbalanced growth in long run equilibrium.

Proposition 4. *During LSGE the physical/human capital ratio, k/h, is increasing; that is,* $\hat{k} - \hat{h} > 0$.

Proof. See Appendix A.

Corollary to Proposition 4*. As* $t \to \infty$, $\hat{h} \to \hat{k}$.

Along the path for sustainable growth, the accumulated asset used in the clean sector (*k*) grows faster than the accumulated asset (*h*) common to both sectors. Along the equilibrium growth path these two growth rates asymptotically converge. However, in finite time, the difference in growth rates has important consequences for the economy. These consequences are:

Proposition 5*. Along the equilibrium expansion path: (i) The share of labor employed in the dirty sector declines. (ii) The ratio of value of dirty output to the value of total output declines. (iii) The physical capital to output ratio in the economy rises.*

Proof: See Appendix A.

Parts *(i)* and *(ii)* of Proposition 5 provide predictions consistent with some of the most robust stylized facts of the modern economic growth process. Along the growth path, the share of employment in the primary (or dirty) sector falls, and that the share of output in the primary sector also declines (Kongsamut et. al. (2001) provide ample evidence on this). Most authors have attributed such structural change primarily to a change in consumer demand toward non-primary goods and/or technological change biased against the primary sector. Here we show that even if demand plays no role (as is the case in a small open economy) and if knowledge growth is not biased to any sector, structural change occurs simply because the primary sector is based on a degradable natural resource which is bounded from above at maximum carrying capacity.

A corollary follows from Proposition 5.

Corollary to Proposition 5. *Labor productivity in the primary sector expands at a rate* $\hat{h}$*, the same rate at which productivity in the clean sector expands.*

Proof: See Appendix A.

The prediction from this corollary is also consistent with the experience of many countries: Primary sectors such as agriculture, forestry and others have experienced rapid growth of output per worker, almost as fast as the non-primary sectors. However growth in output per worker has not prevented the fall in the primary sectors share in GDP. The reason has mostly been the migration of labor from the primary sectors. Consider the experience of the United States over the period 1987-2000. Faruqui et.al. (2003), for example, estimate the annual labor productivity growth of the primary industries in the USA at 3.1% while that of the manufacturing sector at 3.3% per year.

The Decentralized Market Equilibrium and the Social Planner's Solution

We have presented the solution of the economic growth problem assuming that a benevolent social planner directs the economy. One could interpret our results as being normative rather than positive. However, it is well known that the social planner solution replicates decentralized market equilibrium under certain conditions (see the discussion in Section 2.4, Barro and Sala-I-Martin (1995), pp 71). In particular, if all assets have well-defined property rights, if all factor and product markets are competitive (that is firms and households are price takers in all markets), and if agents have rational expectations, a market economy replicates the planner's solution.

However, for the context of a sustainable growth model, the assumption that well-defined property rights exist for the natural resource might be unrealistic. This is especially true for developing countries. Some might argue that in spite of ill-defined property rights on the natural resource, as long as the government sets optimal environmental policy,[16] the planner's solution can still be interpreted as the private decentralized outcome. However, such policies are not always easy to implement, or enforce. Thus, in order to extend the scope of our analysis, we now turn to the case where property rights on the natural resource are ill-defined and the government (or planner) is unable or unwilling to implement optimal environmental policy.

IV.- Open Access

In this section we consider the case where the natural resource is harvested under open access. Note that under open access, if the planner does not intervene to internalize environmental costs, the economy completely ignores the long run implications of extraction or investment.

Formally, this implies that the shadow value of the natural resource, η is considered to be equal to zero by economic agents. There is no investment in natural capital anymore. The disregard for future implications of harvest from, and investment in, the natural asset imply that the net marginal value products of man-made assets are not equalized to the net marginal contribution of natural capital (equation (26) ceases to hold). In this case, if LSGE exists, it is ruled by conditions quite different from those presented earlier.

Under open access, equation (28) becomes,

$$(h(\bar{L}-l_c))^o = \frac{g(n^o)}{\phi n^o}, \tag{31}$$

[16] This would require the government to set environmental regulation which induces the private sector to internalize resource externalities, and subsequently invest in the public environmental good optimally.

where the superscript "o" denotes the solution under open access. Further, equation (25) representing the equalization of the marginal returns to labor between the clean and dirty sectors also changes. Now the marginal product of labor in the dirty sector does not account for the fact that an extra unit labor in such sector causes a loss of natural capital,

$$F_2((k/hl_c)^o, 1) = pn^o \,. \tag{32}$$

However, equation (27) showing the equalization of the net marginal products of physical and human capital remains exactly the same.

The above new equations complete the characterization of long-run equilibrium under open access to natural resources. Based on these equations we have the following proposition.

Proposition 6. *In long-run equilibrium under open access we have that: (i)* $(k/hl_c)^o = (k/hl_c)^*$*; (ii)* $n^o = [(p-\phi)/p]n^* < n^*$*; (iii) The rate of growth of the economy is not affected by the open access condition,* $\hat{x}_c^* = \hat{x}_c^o$ *and* $\hat{x}_d^* = \hat{x}_d^o$

Proof: See Appendix A.

Proposition 6 shows that sustainable development is possible even when the natural resource is harvested under open access. Equivalently, sustainable development is possible even in the absence of environmental policy.[17] Moreover, the rate of growth of the economy over the long-run is not affected by the environmental inefficiency. However, we find that the long run stationary level of the natural resources is lower under open access, than the case where property rights are complete. It can also be shown that the steady state level of welfare is necessarily lower under open access.

What about the feasibility conditions for LSGE under open access? Clearly the conditions for positive growth and a well defined optimum, parts *(a)* and *(b)* of Proposition 3, are equally

[17] The common factor in these two interpretations is that the value of the environment is not considered in economic decisions, and there is no investment in the protection of the natural resource.

feasible under open access and under the benign planner solution. This follows directly from the fact that $pn^o = (p-\phi)n^*$ as shown in Proposition 6. Conditions *(c)* and *(d)* from Proposition 3 are no longer necessary in open access. The lack of relevance of condition *(c)* means that the survival of the clean sector is now assured solely by the Inada conditions.

Condition *(e)* in Proposition 3, is easier to satisfy with resource open access than under the social planner. This condition under open access becomes $F_2((k/hl_c)^o,1) < p\overline{n}$ instead as $F_2((k/hl_c)^*,1) < (p-\phi)\overline{n}$ under the planner's solution. The left hand sides in the above expressions are equal but under open access assuring the permanence of the dirty sector over the long-run requires a lower price level. Proposition 7 below summarizes the results of comparing open access with the planner's solution.

Proposition 7. *(i) The feasibility conditions for the existence of positive growth over LSGE are not more stringent under open access to natural resources than under optimal natural resource policy; (ii) under open access it is more likely that the primary sector will be able to survive as a productive sector; (iii) Under open access, and in contrast with the case of well-defined property rights, the Inada conditions are sufficient to assure the survival of the clean sector.*

The results shown in Propositions 6 and 7 are quite striking. Not only lack of property rights on the natural resource and the lack of investment in the natural resource do not affect the potential rate of growth of the economy but also it does not affect the likelihood that the economy achieves sustainable economic growth. That is, optimal environmental policy is not a necessary condition for LSGE. Neither is a sufficient condition as shown in Proposition 3.

V. – Transition to the Steady State

We consider initial conditions that generally characterize a developing country: relative scarcity of human capital, and an excess of environmental resources. Formally this implies that $n_{00} > n^*$ and $k_{00} > (1/c_1)[\overline{L}h_{00} - \chi(n^*)/(p-\phi)]$ (please see equation (30) for the context of this latter inequality).

The open access case. The best way to understand the transitional dynamics is to start from the simplest possibility. Assume there is open access to the environmental resource. Open access in the resource sector implies that there are no investment in the environment either during transition or in equilibrium. The initial condition of excess physical capital relative to human capital implies that during transition there is investment only in human capital: the asset with the higher returns. Once equilibrium is established, returns to man-made assets are equalized and equilibrium, characterized by investment in both man-made assets, is achieved.

Equilibrium in the natural resource happens concurrently with the equalization of the return to man-made assets, because the only level of the environment consistent with equality of returns to man-made assets is the steady state level (see Equations (31) and (32)). During transition, the excess extraction reduces the stock of the resource, which in turn diminishes the returns of labor in the primary sector, and forces labor to migrate to the industrial sector; a process that continues until equilibrium. Note that this is an innocuous positive description; all that is implicitly assumed is a working and frictionless labor market and no externalities in the markets for human and physical capital. Furthermore, the stability of equilibrium is also guaranteed by the market; for example if production of the primary good increased, the stock of the resource would decline, reducing returns to labor in the primary sector, forcing labor to move into the industrial sector, which will reestablish equilibrium.

Well-defined property rights. Now we turn to the case in which there is no externality in the natural asset. First we note that during transition, the following proposition holds.

Proposition 8. *During transition there can be investment in only physical or human capital, but not in both.*

This proposition can be proved by contradiction. Assume there is investment in both man-made assets during transition. That means that returns of both assets are equalized:

$$\lambda F_1(\cdot) - \Omega\delta_k = F_2(\cdot) l_c + (\lambda p - \eta\phi) n(\bar{L} - l_c) - \mu\delta_h \qquad (33).$$

During transition, as in equilibrium, returns to labor, i.e. wages, must equalize across sectors if labor is free to move, thus, equation (19) also holds during transition. Additionally, if there is investment in both man-made assets we have that $\lambda = \mu = \Omega$; this means that during transition equality in the returns to man-made assets, as in equilibrium, implies (27): $F_1(\cdot) - \delta_k = F_2(\cdot)\overline{L} - \delta_h$. However, since (k/hl_c) is changing during transition, this would disrupt the equality of returns of accruable assets and therefore the investment in both assets at the same time. To close this proof we note with the aid of Figure 1 that (k/hl_c) is not fixed during transition. If it was, then it would mean that the economy is at some point of the horizontal line $(k/hl_c)^*$ other than the equilibrium point at n^*. Points to the right of this equilibrium are discarded because they violate the Kuhn-Tucker condition ($\lambda > \eta$); points to the left would require investment in the three assets at a point outside of equilibrium (see next Proposition) which by definition is not possible, only in equilibrium there is investment in the three assets.

In the presence of a relative surplus of natural resources, one expects a movement of labor into the dirty sector to extract the excess natural resource as fast as possible. However, specialization in the dirty sector is not possible. The surplus of natural resources provides higher returns to labor in the dirty sector, but as labor leaves the clean sector, this also brings about an increase of the labor productivity in the clean sector. Given Inada conditions, wages equalize across the two sectors before specialization can occur. The equality of the value of marginal product of labor across clean and dirty sectors, as explained above, holds throughout the transition (equation (19) always holds).

However, given the surplus of natural resources, there is no investment in natural capital during transition. All investment is allocated to the man-made asset with the highest returns. Given the relative scarcity of *h*, its value of marginal product, $\lambda F_2(\cdot)\overline{L} - \mu\delta_h$, is higher than the value of marginal product of *k*, $\lambda F_1(\cdot) - \Omega\delta_k$. Therefore along the transition there is investment only in *h*.

This sensible economic rule of investing in the man-made asset with the highest returns is also the mathematical requirement for convergence of the multipliers. If there is initial investment in h we know that $\lambda = \mu > \eta, \Omega$. For there to be convergence, μ has to fall faster than the shadow value of the other assets. Investment lowers the scarcity of this asset relative to the other assets, and there is eventual convergence.

The convergence path described is shown in Figures 1 and 2. In Figure 1, the dotted line starting from point A, represents the saddle path towards steady state in the $((k/hl_c), n)$ space. We know that this path lies above the solid diagonal line which represents the equilibrium equality of net marginal product of labor in both sectors. This is because the Kuhn-Tucker conditions for the non-negativity of investment require that $\lambda > \eta$. Moreover, it is easy to see that because $\lambda > \eta$ the dotted line in Figure 1 must be steeper than the solid line too.

Note that point A is always attainable for any initial endowment of k and h, as l_c instantaneously adjusts to allow the economy to start at converging point A. The transition path as described by the figures is characterized by growing h (the asset where investments are made); falling k as there is no investment in the asset; falling k/hl_c ratio; and falling n, which is in surplus. Although not seen in the graph, l_c will start low and will increase during the transition. This is due to the fact that the returns to labor in the dirty sector relative to the clean sector $((p-(\eta/\lambda)\phi)n)$ fall as a consequence of both the decline in n, as well as the rise in the relative shadow value η/λ.

In Figure 2 we observe that the assumed relative scarcity of h implies that initially the economy is in a point like A, to the left of the diagonal line that represents equilibrium between man-made and natural assets. The transition is described as accumulation of h, while k declines due to depreciation of stocks. Equilibrium is pictured as the diagonal line, to which the economy arrives at the same time that equilibrium is established in the natural resource.

Note that the transition path described above is feasible under very general conditions, not only in the neighborhood of steady state. The path would not be feasible though, if there is not enough labor in the economy to initially extract more resources than naturally are regenerated, or if there was an extreme scarcity of man-made assets which would yield an effective labor unable to reduce the surplus of natural resources.

Lastly we consider the case when the economy starts from a position of relative scarcity of the natural resource, $n_{00} < n^*$. In this case the following proposition can be shown:

Proposition 9. *When approaching equilibrium from points described by $n_{00} < n^*$, there will always be investment in the natural asset before steady state is attained.*

This result follows from the fact that there is no discrete jump in labor from transition to equilibrium; labor in the resource sector smoothly adjusts to its equilibrium level. The reader can confirm this equalizing (20) and (22), and solving for labor in the resource sector. Therefore, if the economy arrived to equilibrium by accumulating natural capital without investing in it, when equilibrium is established and investment in the resource starts, the resource would grow ($\dot{n} > 0$), violating the equilibrium assumption. Then, although not necessarily at the beginning of transition, at some point before equilibrium is established, investment in natural capital must start, when $n_{00} < n^*$.

VI.- Conclusion

In this paper we show that structural change allows an open economy to grow with environmental sustainability. Endogenous structural change is a response to the presence of the constraint on natural capital. Structural change takes place even if consumer preferences are homothetic and if technological progress does not discriminate against the primary or dirty sector. In our model, endogenous labor-augmenting productivity growth benefits both the resource extracting and clean sector equally. However, despite this assumption, the model predicts that the primary sector remains stagnant over the long-run. In contrast, the clean sector grows continuously along equilibrium. This growth allows the clean sector to attract an increasing volume of labor (or other inputs used in both sectors) from the primary sector.

Perhaps the most striking result in this paper is that property rights on the natural resource (or optimal environmental policy) are neither necessary nor sufficient for environmentally sustainable economic growth. We have shown that the rate of growth of the economy over the long run is not affected by environmental policy. More importantly, the feasibility of environmental sustainability with positive long-run growth is not hampered by incomplete property rights. Better property rights allow the economy to attain a larger stock of natural resources, and achieve a higher level of consumption and welfare. The real determinant of the feasibility of sustainable growth is the development of a sufficiently productive clean sector that can successfully compete with the primary sector for factors of production and a sufficiently rapid pace of accumulation of knowledge that enables labor-augmenting productivity and structural change. Thus, a necessary condition for sustainable growth is that the true shadow value of knowledge and physical capital be fully reflected in the asset accumulation decisions. Market failures that affect, for example, the ability of the economy to invest in knowledge, may frustrate in the long-run structural change and ultimately the capacity of the economy to achieve both positive growth and environmental sustainability.

Note that the openness of the economy plays an important role in permitting the continuously increasing consumption demand for the dirty goods (caused by the growth of income) to be satisfied by increasing imports or falling exports of the dirty good. That is, economic openness allows the economy to free ride on the rest of the world allowing to reconcile increasing consumption of the dirty good with a stagnant domestic production. In this respect the fixity of the relative price of the dirty good (due to the assumption of the country being small in the world) may play an important role.

References

Aghion, P. and P. Howitt (1998), *Endogenous Growth Theory*, MIT Press, Cambridge.

Arrow, K. J., and G. Debreu, (1954) "Existence of an Equilibrium for a Competitive Economy," *Econometrica*.

Barro, R. and X. Sala-I-Martin (1995), *Economic Growth*, McGraw-Hill, Inc., New York.

Baumol, W. J., S. A. B. Blackman, and E. N. Wolff (1985), "Unbalanced Growth Revisited: Asymptotic Stagnancy and New Evidence," *American Economic Review*, 75(4):806-17.

Baumol, W. J (1967), "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis," *American Economic Review*, 57(3):415-426.

Bovenberg A.L. and S.A. Smulders (1995), "Environmental Quality and Pollution-Augmenting Technological Change in a Two Sector Endogenous Growth Model," *Journal of Public Economics*, 57:369-91.

Bovenberg A.L. and S.A. Smulders (1996), "Transitional Impacts of Environmental Policy in an Endogenous Growth Model," *International Economic Review*, 37(4):861-893.

Brander J. and M. Taylor, (1998) "The Simple Economics of Easter Island: A Ricardo-Malthus Model of Renewable Resource Use," *American Economic Review*, 88(1): 119-138.

Brander J. and M. Taylor, (1998a) "Open Access Renewable Resources: Trade and Trade Policy in a Two-Country Model," *Journal of International Economics*, 44(2):181-209.

Brock W. A, and M. S. Taylor (2004), "Economic Growth and the Environment: A Review of Theory and Empirics," *NBER Working Paper Series, WP 10854*, Cambridge MA.

Brown G. (2000) "Renewable natural resource management and use without markets", *Journal of Economic Literature*, 38, 875-914

Cass, David (1965), "Optimum Growth in an Aggregative Model of Capital Accumulation," *Review of Economic Studies*, 32: 233-240.

Chenery, H. B. (1960), "Patterns of Industrial Growth," *American Economic Review*, 50:624-654.

Clark, Colin (1990), Mathematical Bioeconomics. The Optimal Management of Renewable Resources, John Wiley and Sons, New York.

Conrad, J.M. (1995), "Bioeconomic models of the fishery," *Handbook of Environmental Economics,* (D. Bromley, ed.) Oxford, Blackwell, 405-432

Diewert W. E. (1974), "Applications of Duality Theory," in *Frontiers of Quantitative Economics* Vol. II, edited by M. Intriligator and D. Kendrick, North-Holland, Amsterdam, pp. 106-71.

Dixit, Avinash and Victor Norman, "Theory of International Trade," Cambridge University Press, Great Britain, 1980.

Echevarria, C. (1997), "Changes in Sectoral Composition Associated with Economic Growth," *International Economic Review*, 38(2):431-52.

Eliasson, L. and S. Turnovsky (2004). "Renewable Resources in an Endogenously Growing Economy: Balanced Growth and Transitional Dynamics" *Journal of Environmental Economics and Management*, 48(3): 1018-1049.

Faruqui, U.; W. Gu; M. Kaci; M. Laroche; and J. Maynard (2003), "Differences in Productivity Growth: Canadian-US business Sectors, 1987-2000." *Monthly Labor Review*, 16-29.

Gordon, H. S. (1954), "The Economic Theory of a Common Property Resource: The Fishery," *Journal of Political Economy*, 63(2):124-42.

Hotelling, H. 1931. "The Economics of Exhaustible Resources", *Journal of Political Economy*, 39, 137-175.

Huang, C. and D. Cai (1994), "Constant-Returns Endogenous Growth with Pollution Control," *Environmental and Resource Economics,* 4(4):383-400.

Kongsamut P., S. Rebelo, and D. Xie (2001), "Beyond Balanced Growth", *Review of Economic Studies*, 68(4):869-82.

Koopmans, Tjalling (1965), "On the Concept of Optimal Economic Growth," in *The Economic Approach to Development Planning*, Amsterdam, North Holland.

Kuznets S. (1957), "Quantitative Aspects of the Economic Growth of Nations II," *Economic Development and Cultural Change*, Supplement to Volume V(4):3-11.

Laitner, J. (2000) "Structural Change and Economic Growth," *Review of Economic Studies,* 67:545-61.

Lucas, R. E. (1988), "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22(1): 3-42.

Mani, M. and D. Wheeler (1998), "In Search of Pollution Heavens? Dirty Industry in the World Economy, 1960-1995", *Journal of Environment and Development*, 7: 215-47.

McAusland, C. (2005), "Learning by doing in the Presence of Open Access Renewable Resource: Is Growth Sustainable?" *Natural Resource Modeling* 18(1): 41-68

Munro G. R. and A. D. Scott (1993), "The economics of Fisheries Management," in *Handbook of Natural Resource and Energy Economics, Volume III*, Kneese and Sweeney eds., North Holland.

Parry, I. (1999), "Productivity Trends in the Natural resource Industries: A Cross-cutting Analysis", in R.D. Simpson (ed.), *Productivity in Natural resource Industries,* Resources for the Future, Washington DC.

Schaefer, M.B. (1957), "Some Considerations of Population Dynamics and Economics in Relation to the Management of Marine Fisheries," in *Journal of the Fisheries Research Board of Canada*, 14:669-81.

Schou P. (2000), "Polluting Non-Renewable Resources and Growth," *Environmental and Resource Economics,* 16(2): 211-27.

Smulders S.A. and R. Gradus (1996), "Pollution Abatement and Long-Term Growth," *European Journal of Political Economy*, 12:505-32.

Smulders, S. (1999), "Endogenous Growth Theory and the Environment," in J.C.J.M. van den Berg (eds.) *Handbook of Environmental and Resource Economics*, Cheltenham: Edward Elgar: 610-621.

Solow, R. M. (1956), "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, 70(1):65-94.

Stokey N.L. (1998), "Are There Limits to Growth?" *International Economic Review*, 39(1):1-31.

Tahvonen O. and J. Kuuluvainen (1991), "Optimal Growth with Renewable Resources and Pollution," *European Economic Review*, 35:650-61.

Appendix

A. Proofs

Proof to Proposition 1. In order to have positive consumption growth forever, growth in at least one asset must always be positive. This implies that either investments in human capital, physical capital or both should be positive in LSGE (note that natural capital cannot grow forever). We consider these two cases separately. Case (1): Positive accumulation of human capital, implies that $I_h > \delta_h h > 0$. From the first order conditions this requires that $\lambda = \mu$. Using equations (18), (19) and (23) consumption growth is given by,

$$\hat{x}_c = \varepsilon\left((p - \frac{\eta}{\lambda}\phi)n\bar{L} - \rho - \delta_h \right). \qquad \text{(A1)}$$

Since, given the definition of LSGE, both $\hat{x}_c$ and n are constant, the above equation implies that η / λ must also be constant. This implies that $\hat{\lambda} = \hat{\mu} = \hat{\eta}$ is necessary for LSGE. Further, (19) can be written as

$$F_2(k / hl_c, 1) = (p - \phi\eta / \lambda)n \qquad \text{(A2)}$$

This equation implies that (k / hl_c) must also be constant in LSGE. The fact that the rate of growth of x_c is constant by definition of LSGE implies that $\hat{\mu}$ and, therefore, $\hat{\eta}$ are both constant. Given the constancy of $\eta / \mu = \eta / \lambda$ and the fact that the resource stock, n, is also constant, (22) implies that the variable $h(\bar{L} - l_c)$ must also be constant over time. Differentiating we get that

$$\hat{h} = [l_c / (\bar{L} - l_c)] \cdot \hat{l}_c \qquad \text{(A3)}$$

which is greater than zero because $I_h > \delta_h h > 0$ by assumption. This means that both h and l_c are increasing over time. But since (k / hl_c) must be constant, we have that $\hat{k} > 0$, which in turn means that $I_k > 0$ and, therefore, $\Omega = \lambda$. Hence, LSGE requires that $\hat{\Omega} = \hat{\lambda} = \hat{\mu} = \hat{\eta}$, in addition to the condition listed earlier.

Case (2): Alternatively, if k is accumulated, then $I_k > \delta_k k > 0$, and we have that $\lambda = \Omega$. In this case a constant growth rate of x_c implies that $\hat{\Omega}$ is constant and negative. From (21) it immediately follows that $F_1(k / hl_c, 1)$ is constant which in turn implies that (k / hl_c) is also constant. Equation, (18'), in turn, means that $\eta / \lambda = \eta / \Omega$ does not change over time. Therefore, $\hat{\Omega} = \hat{\eta}$. Equalizing (21) and (22) makes it clear that as in case (1), $h(\bar{L} - l_c)$ is also constant over time. Therefore, we also have that (A3) holds. Now using the latter expression and the fact that (k / hl_c) is constant which means that: $\hat{k} = \hat{h} + \hat{l}_c$, we obtain that $\hat{k} = \left(1 + [(\bar{L} - l_c) / l_c]\right) \cdot \hat{h}$. Given that $\hat{k} > 0$, we have that $\hat{h} > 0$ and, therefore, $I_h > 0$ which, in turn, means that $\mu = \Omega = \lambda$. Hence as in case (1) $\hat{\Omega} = \hat{\lambda} = \hat{\mu} = \hat{\eta}$ is implied. ■

Proof to Proposition 2. Suppose that the non-negativity constraint is binding. That is, $I_n = 0$ while in fact the unrestricted optimum would require $I_n^* < 0$. Use a tilde to define the solution of the system (25) to (28) when $I_n = 0$ and a double star to denote an hypothetical solution that allows for an unrestricted optimum, I_n^{**}, which is less or equal to zero if $I_n = 0$. We now compare the restricted and unrestricted (hypothetical) solutions. In this case condition (28) is satisfied if $\tilde{n} < n^{**}$ or $\widetilde{hl}_d < (hl_d)^{**}$, or both (where $l_d = \bar{L} - l_c$). If $\tilde{n} < n^{**}$ then by (25) $\tilde{r} < r^{**} = 1$. Also, from (26) it is clear that since $g_n(n)$ is decreasing in n that $\widetilde{hl}_d < (hl_d)^{**}$. Using analogous reasoning, it is clear that if $\widetilde{hl}_d < (hl_d)^{**}$ then $\tilde{n} < n^{**}$ as well. That is, we need both inequalities to hold simultaneously.

We now show that these two inequalities together are inconsistent with the first order conditions when evaluated at LSGE. Assume that $\tilde{n} < n^{**}$ and $\widetilde{hl}_d < (hl_d)^{**}$. The solution when $I_n = 0$ implies

$$\widetilde{hl}_d = g(\tilde{n}) / \phi\tilde{n} \tag{A4}$$

Also, since $I_n^{**} < 0$ by assumption we can define a variable $v = -I_n^{**} / (\phi n^{**}) > 0$ such that,

$$(hl_d)^{**} + v = g(n^{**}) / (\phi n^{**}) \tag{A5}$$

Given that $g(n)/n$ is decreasing in n, we have that $g(n^{**}) / (\phi n^{**}) < g(\tilde{n}) / (\phi\tilde{n})$. Now subtracting (A5) from (A4) we obtain,

$$\widetilde{hl}_d - hl_d^{**} = g(\tilde{n}) / (\phi\tilde{n}) - g(n^{**}) / (\phi n^{**}) + v > 0,$$

which means that $\widetilde{hl}_d > hl_d^{**}$ thus contradicting the earlier supposition. Hence either $\tilde{n} > n^{**}$ **or** $\widetilde{hl}_d > hl_d^{**}$. But $\tilde{n} > n^{**}$ means that $\tilde{r} > r^{**} = 1$ according to (25), which is inconsistent with the Kuhn-Tucker condition (18). The other option, $\widetilde{hl}_d > hl_d^{**}$ is not consistent with the condition (26) when $\tilde{n} \le n^{**}$. Hence, the only feasible equilibrium is one with $I_n^{**} > 0$ and $r = 1$.■

Proof to Proposition 4. Taking time derivatives of(30):

$$\hat{h} = \hat{k}\left[1 - \frac{\chi(n^*)}{(p-\phi)h\bar{L}}\right]$$

which given that $0 \le \chi \le (p-\phi)h\bar{L}$ then $\hat{k} \ge \hat{h}$, and that as h grows, so does k.■

Proof to Proposition 5. The first two parts of Proposition 5 are derived from the fact that the total extraction effort $(hl_d)^*$ as well as the level of n remain constant along the equilibrium growth path while the clean sector continuously grow as k, h and l_c all expand. This leads to (*ii*). The fact that hl_d is constant while h continuously grows in LSGE means that l_d must decline over time. Hence, result (*i*) follows. The fact that the physical capital to output ratio is rising in the economy is not obvious but may be shown by looking at the definition:

$$\frac{k}{y} = \frac{k}{pnh(\bar{L} - l_c) + F(k, hl_c)} = \frac{1}{(h/k)pn(\bar{L} - l_c) + F_1(\cdot) + (h/k)l_c F_2(\cdot)},$$

where in the second equality we divided by k and expanded clean output using the property of homogeneity of the clean technology. If we replace $l_c = (k/h)\cdot c_1$, we get:

$$\frac{k}{y} = \frac{1}{(h/k)pn\overline{L} - pnc_1 + F_1(\cdot) + c_1 F_2(\cdot)}. \qquad \blacksquare$$

Proof to Corollary to Proposition 5. Output per worker in the primary sector is hn. Since along LSGE n is constant at n^*, labor productivity increases at a rate $\hat{h}$. Output per worker in the dirty sector is $F(k, hl_c)/l_c = F(k/l_c, h)$. Where the equality follows from the fact that F is linearly homogenous in k and hl_c. Since $F(k/l_c, h)$ is homogenous of degree one in k/l_c and h we have that $F(k/l_c, h) = h\ F(k/hl_c, 1)$. Noting that k/hl_c is constant along LSGE, we have that output per worker in the clean sector also grows at a rate $\hat{h}$.■

Proof to Proposition 6. Part *(i)* follows from the fact that condition (27) is valid in open access as well. Under open access the shadow values of h and k are also equal and, therefore, they must decline at an identical and constant rate, $\hat{\mu} = \hat{\Omega}$. This means that $F_1 - \delta_k = F_2 l_c - \delta_h + n(\overline{L} - l_c)$. Using (31) in the above expression we obtain a condition identical to (27). That is, $(k/hl_c)^o = (k/hl_c)^*$. Part (*ii*) follows by noting that $F_2((k/hl_c)^*, 1) = F_2((k/hl_c)^o, 1)$ (as equation (27) remains the same) and $\phi > 0$. From (25) and (32) we have that $pn^o = (p-\phi)n^*$ which shows part (*ii*). Part (*iii*) is shown by using (24) noting that $\hat{\lambda} = \hat{\mu} = \hat{\Omega}$. Thus,

$$\hat{x}_c = -\varepsilon\hat{\lambda} = \varepsilon\left(F_2((k/hl_c)^o, 1)\overline{L} - \delta_h - \rho\right) = \varepsilon\left(F_2((k/hl_c)^*, 1)\overline{L} - \delta_h - \rho\right).\blacksquare$$

B. Solution to LSGE:

We proceed to find explicit solutions to the variables of our model in equilibrium using explicit functions for the clean technology (3) and the natural growth function (6). First, equating $\hat{\mu}$ and $\hat{\Omega}$, we find that the equality of returns to man-made assets imposes a fixed (k/hl_c) ratio, assuming for notation simplicity that depreciation rates are equal among man-made assets $(\delta_h = \delta_k)$:

$$\left(k/hl_c\right)^* = \alpha/[(1-\alpha)\overline{L}] \qquad \text{(A6).}$$

This last expression immediately provides a solution for the labor demand in the clean sector:

$$l_c(t) = k(t)/h(t)\cdot[(1-\alpha)\overline{L}/\alpha] \qquad \text{(A7).}$$

Using this optimal $(k/hl_c)^*$ ratio, and the labor market equilibrium condition (19), we can find the fixed optimal level of natural resources:

$$n^* = \frac{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(p-\phi)\overline{L}^{\alpha}} \qquad \text{(A8)}$$

Equating $\hat{\Omega}$ and $\hat{\eta}$, we find that effective extraction effort must be:

$$h(\overline{L} - l_c) = hl_d = [(p-\phi)n^*\overline{L} - \gamma(1 - 2n^*/\overline{n}) - \delta]/(p-\phi) \qquad \text{(A9),}$$

which is fixed during LSGE as n must remain fixed too. To reduce the notation clutter we use again χ as a label for the constant in (A9):

$$\chi(n^*) \equiv (p-\phi)n^*\overline{L} - \gamma(1 - 2n^*/\overline{n}) - \delta.$$

Using (A7), we can find the relation between *k* and *h* that must hold during steady state:

$$k(t)=[\alpha/(1-\alpha)]\left\{h(t)-\chi(n^*)/[(p-\phi)\bar{L}]\right\} \quad \text{(A10)}$$

Given that during equilibrium there is a fixed relationship between *k* and *h* during LGSE, we proceed to express the rest of the variables in terms of one of them, by choice, *k*. We begin with output: using (A7) and (A9), we express dirty output as:

$$y_d = n^*\chi(n^*)/(p-\phi) \quad \text{(A11);}$$

and using (A6), we can express clean output as,

$$y_c(t)=A[(1-\alpha)\bar{L}/\alpha]^{(1-\alpha)}\cdot k(t) \quad \text{(A12).}$$

We continue with investment. Using the fact that *n* remains fixed during LSGE, and (7) we express investment in the resource as:

$$I_n=\phi n^*\chi(n^*)/(p-\phi)-g(n) \quad \text{(A13).}$$

Of course from, (8) investment in physical capital is $I_k(t)=\dot{k}(t)-\delta k(t)$. Similarly, but using (A10) and $\dot{k}=[\alpha/(1-\alpha)]\dot{h}$ obtained by differentiating (A10), we can express, investment in human capital in terms of *k*:

$$I_h(t)=[(1-\alpha)/\alpha]\dot{k}(t)+[(1-\alpha)/\alpha]\delta k(t)+\delta\chi(n^*)/[(p-\phi)\bar{L}] \quad \text{(A14).}$$

So finally, using (A11) - (A14) we can express the budget constraint in terms of only one variable:

$$x_c+px_d=\left[A\left(\frac{(1-\alpha)\bar{L}}{\alpha}\right)^{(1-\alpha)}-\delta/\alpha\right]\cdot k(t)-\frac{1}{\alpha}\cdot\dot{k}(t)+\left[n-\frac{\delta}{(p-\phi)\bar{L}}\right]\chi(n^*)+g(n^*) \quad \text{(A15).}$$

We have almost completely solved the model in equilibrium, all we need is an expression for *k*(*t*). We need to guess a functional form for *k*(*t*). Before doing so we "educate" our guess. We know that at infinity both man-made assets grow at the same and constant rate. From Appendix C, we know that this rate approximately equals the rate of growth of consumption. Additionally, we know that the rate at which *k* grows has to be such, that it allows for a constant, known, rate of growth of consumption. With all this information we propose and confirmed that:

$$k(t)=k_0\cdot e^{\omega(t-t_0)}-b \quad \text{(A16),}$$

where k_0 is the level of physical capital at the time steady state was achieved, t_0 is the time at which equilibrium was attained, and ω is the rate of growth of consumption: $\omega\equiv-\varepsilon\cdot\hat{\mu}$, and b is an unknown constant. In general, with positive depreciation rates, k_0 is unknown, however, if depreciation of capital is zero, and transition is characterized by investment *h*, then $k_0=k_{00}$; or if transition is characterized by investment in *k* and depreciation in *h* is zero then $k_0=[\alpha/(1-\alpha)]\left\{h_{00}-\chi(n^*)/[(p-\phi)\bar{L}]\right\}$.

To find the constant *b* we express consumption, using (14) into (A15) as:

$$x_c=\xi M\cdot k(t)-\xi/\alpha\cdot\dot{k}(t)+\xi N \quad \text{(A17),}$$

where the constants $M=\left[A[(1-\alpha)\bar{L}/\alpha]^{(1-\alpha)}-\delta/\alpha\right]$ and $N=[n-\delta/((p-\phi)\bar{L})]\chi(n^*)+g(n^*)$ are used to manipulate the equation. Since the rate of growth of consumption is known, we differentiate (A17), using the fact that (A16) implies that $\hat{k}(t)=\omega+\omega b/k(t)$:

$$\hat{x}_c = \dot{k}(M - \omega/\alpha)/[k(M - \omega/\alpha) + (N - \omega b/\alpha)] = \omega \qquad \text{(A18)},$$

We can use (A18) to solve for b:

$$b = \frac{N}{M} = \frac{[n^* \chi(n^*) + g(n^*)](p - \phi)\alpha \bar{L} - \alpha\delta\chi(n^*)}{(p - \phi)\bar{L}[A((1-\alpha)\bar{L})^{(1-\alpha)}\alpha^{\alpha} - \delta]} \qquad \text{(A19)}.$$

Note that b is positive, it easily checked by noting that both M and N are greater than zero. M must be positive in a growing economy, and N must be positive if returns to man-made assets are positive. Thus, the growth rate of capital converges to ω from above.

This concludes the representation of our model. With $k(t)$ known and as a function of time exclusively, we can represent the other variables of our model:

$$h(t) = [(1-\alpha)/\alpha] \cdot k_0 e^{\omega(t-t_0)} + \chi(n^*)/[(p-\phi)\bar{L}] - [(1-\alpha)/\alpha]b \qquad \text{(A20)}$$

and

$$l_c(t) = \frac{(p-\phi)(1-\alpha)\bar{L}^2}{(p-\phi)(1-\alpha)\bar{L} + \alpha\chi(n^*)/k(t)} \qquad \text{(A21)}.$$

Note how $l_c(t)$ converges as time tends to infinity to $\bar{L}$.

We can finish by re-expressing consumption as function of time only:

$$x_c(t) = \xi \left\{ \left[A\left(\frac{(1-\alpha)\bar{L}}{\alpha} \right)^{(1-\alpha)} - \frac{(\delta + \omega)}{\alpha} \right] \cdot k(t) + \left[n^* - \frac{\delta}{(p-\phi)\bar{L}} \right] \chi(n^*) + g(n^*) - \frac{\omega b}{\alpha} \right\} . \text{(A22)}$$

C. Conditions Imposed by the Transversality Condition

In order to check the transversality condition, we need some idea about how assets are accumulated towards the end of the planning horizon, i.e. infinity. To get an approximation, we begin by taking a time derivative of the budget constraint (10),

$$\hat{x}_c x_c + \hat{x}_d p x_d = pnhl_d(\widehat{hl_d}) + kF_1(\cdot)\hat{k} + hl_c F_2(\cdot)(\widehat{hl_c}) - I_n \hat{I}_n - I_k \hat{I}_k - I_h \hat{I}_h \qquad \text{(A23)}.$$

Since the (k/hl_c) is constant in steady state, we have that $\hat{k} = (\widehat{hl_c})$, also recall that both I_n and hl_d are also constant in equilibrium, and that both consumption goods grow at the same rate. Furthermore, from Proposition 3 we know that at infinity both k and h grow at the same rate, therefore both investment rates must grow at the same rate at infinity, and that rate must be equal to the rate of growth of man-made assets. Thus, using the fact that the clean technology is homogenous of degree 1 we can rewrite (A23):

$$\lim_{t\to\infty} : \hat{x}_c(x_c + px_d) = \hat{k}(F(k, hl_c) - I_k - I_h) \qquad \text{(A24)}.$$

Also, towards infinity we have that:

$$\lim_{t\to\infty} : \frac{x_c + px_d}{F(k, hl_c) - I_k - I_h} \approx 1,$$

because the difference between numerator and denominator is a constant, $pnhl_d - I_n$, and its ratio over $(F(k, hl_c) - I_k - I_h)$ tends to zero as clean output grows. Thus, we have that at infinity:

$$\lim_{t\to\infty} : \hat{x}_c \approx \hat{k} \equiv \omega(n^*) = \varepsilon\left[(p-\phi)n^*\overline{L} - \delta_h - \rho\right] \qquad \text{(A25)}.$$

The right hand side of (A25) contains the rate of growth of x_c from (24).

On the other hand, the rate of fall of Ω may be readily obtained from (19'):

$$\Omega(t) = \Omega_0 e^{-\left[(p-\phi)n^*\overline{L}-\delta_h-\rho\right](t-t_0)} \qquad \text{(A26)},$$

where Ω_0 is the level of the shadow values when steady state was achieved (at time t_0), and where we also used the fact that $\hat{\Omega} = \hat{\mu}$. We can now express the transversality condition for *k* (23):

$$\lim_{t\to\infty} : \exp[-\rho t]\cdot k_\infty \exp\left[\varepsilon\left[(p-\phi)n^*\overline{L} - \delta_h - \rho\right]t\right]\Omega_0 \exp\left[-\left[(p-\phi)n\overline{L} - \delta_h - \rho\right](t-t_0)\right] = 0 \qquad \text{(A27)},$$

where k_∞ is a level of physical capital accumulated when time approaches infinity. From (A27), it can be shown that this condition will hold when,

$$\rho > (\varepsilon - 1)\left[(p-\phi)n\overline{L} - \delta_h - \rho\right] \qquad \text{(A28)}.$$

Note that the right hand side of (A28) is equal to $\hat{U}$, so condition (A28) is similar to most endogenous growth models. Also note that (A28) can be re arranged like in condition *(b)* of Proposition 3. Given that both man-made assets grow at the same rate towards infinity, it is easy to show that condition (A28) also applies also for *h*. On the other hand, the transversality condition always holds for natural capital, as this asset is fixed in equilibrium, as long as $\hat{\lambda} < 0$.

D. Stagnation when the Society Specializes in the Dirty Sector.

Let us formally illustrate stagnation in the case where society specializes in the dirty sector. The only possible avenue for growth is accumulation in human capital. However, as *h* expands, in order to keep *n* constant, investment in *n* also expands. This implies that I_n and I_h need to be both positive, and that $\eta = \mu$, and hence, $\hat{\eta} = \hat{\mu}$. From (20) and (22) noting that the production function *F* now plays no role in (20) we obtain

$$(p-\phi)n\overline{L} - \delta_h = (p-\phi)h\overline{L} + g_n(n)$$

That is the net marginal product of *h* must be equal to that of n. Since n is bound by the carrying capacity this means that in long-run specialized equilibrium *h* and n must reach stationary values. That is, positive growth in the long-run, and hence LSGE, are unattainable if the society specializes in the dirty sector.

Figure 1

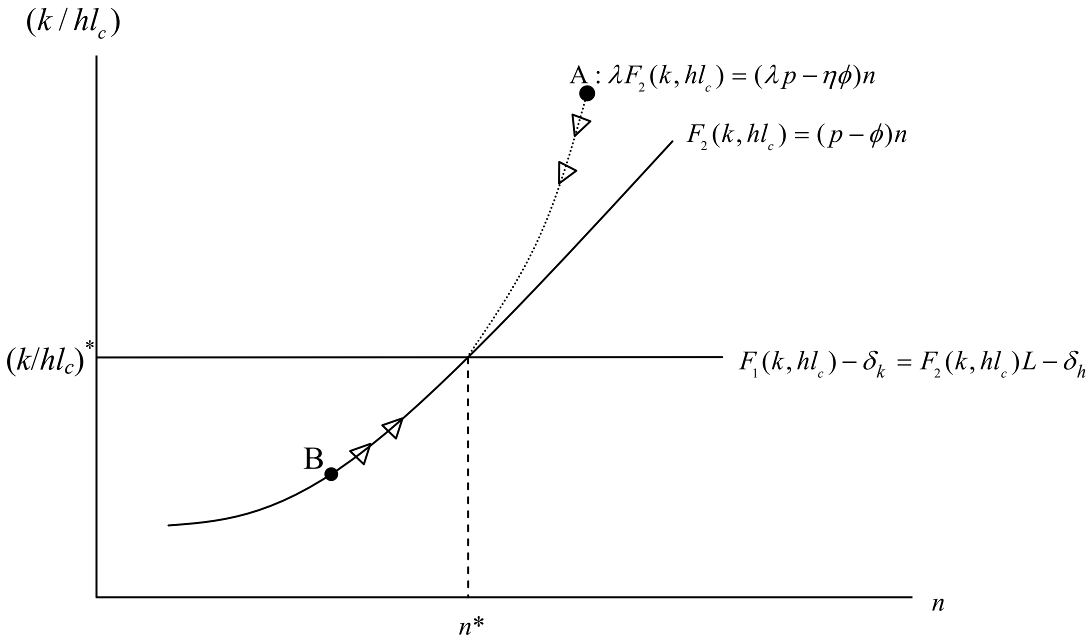

Figure 2